EXAM 09-03-2026

FROM ATOMS TO SOLIDS. # QUESTIONS: 3

The grade is calculated by: GRADE=1+(TotalPoints/10)

If needed, you can use the following: $m_e/M_p = 1/1836$; $\alpha = 1/137$; $hcR_\infty = -13.7$ eV; R_∞ =109737.3 cm^{-1}.

1. Many-electron systems and approximations (33 points)

Consider the calcium atom, Ca, with a configuration of $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$.

a. (6 points) Write down the Hamiltonian of this atom. You may use either SI or atomic units.

b. (6 points) Explain the different terms in the Hamiltonian.

c. (6 points) To calculate energies and wave functions of many-electron atoms several approximations are made. Describe briefly (2-3 sentences) the main assumption(s) underlying the Central Field Approximation.

d. (5 points) Why do we usually solve the Central Field problem iteratively?

The ionization potential of Ca^+ in the ground, $1s^2 2s^2 2p^6 3s^2 3p^6 4s$ state is 11.872 eV, while its ionization potential in the $1s^2 2s^2 2p^6 3s^2 3p^6 4d$ state is 4.824 eV.

e. (6 points) Calculate the quantum defects for both orbitals, δ_{4s} and δ_{4d}.

f. (4 points) Explain the difference between these quantum defects.

2. One-electron systems (26 points)

a. (6 points) The spectrum of a one-electron ion of some element shows that its $3s$ orbital is at 14070906.59 cm^{-1} relative to the ground state. Which element is this (the atomic number Z is a sufficient answer)?

b. (6 points) For the n=3, l=2 level, what will be the size of the splitting in this ion due to the fine structure (use either eV or cm^{-1})? If you have not derived the atomic number in the previous sub-question, express your answer in terms of Z.

c. (8 points) Sketch schematically the energies of all the states with n=3, relative to non-relativistic energy, including the spin-orbit splittings and other fine structure effects for the relativistic levels. Explain your sketch.

d. (6 points) Predict the wavenumber of the Lyman-α transition in muonic hydrogen, which is an exotic atom consisting of a proton orbited by a negative muon ($m_\mu = 207 m_e$, charge: e).

3. LS coupling and transitions in many-electron atoms. (31 points)

In the first part of this question, we are considering the element strontium, which has a $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 5s^2$ ground state.

a. (4 points) Which term corresponds to this ground state? What is the parity of this configuration?

b. (8 points) Consider an electronically excited state where one of the $5s$ electrons is promoted to the $4d$ orbital. Which terms (and levels) arise from this excited configuration?

c. (5 points) consider the configuration where one of the $5s$ electrons is promoted to the $5p$ and giving rise to ${}^1P_1$, ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$ levels. Which of these excited levels will have the shortest lifetime? Explain your answer.

d. (6 points) For an isotope with a non-zero nuclear spin I, the ${}^1P_1$ level will be split further by the magnetic hyperfine interaction. For ^{81}Sr, this level splits into 2 states, while for ^{87}Sr it splits into 3 states. What are the nuclear spins of ^{81}Sr and ^{87}Sr?

Consider Dy, with the ground state configuration of [Xe]$6s^2 4f^{10}$.

e. (8 points) Determine the ground term and the ground level of Dy. Explain your solution steps.